Precalc Warm Up # 11-4

1. $1 + 3x + 9x^2 + \ldots = 10$ Find x

2. Kara and Dave bought a house in January of 1983 for $240,000. It's value goes up 4% each year. What is it worth at the beginning of 2007?

From Monday's assignment, p. 262

8th term

7. The Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, . . . in which each term is the sum of the previous two terms is neither arithmetic nor geometric. However, after the eighth term (21) the sequence becomes approximately geometric. If we assume that the sequence is geometric:

(a) What is the common ratio of the sequence (to four significant figures)?

(b) Assuming that the Fibonacci sequence can be approximated by the geometric sequence after the eighth term, what is the approximate sum of the first 24 terms of the Fibonacci sequence?

a)

n	8	9	10	11	12	13
	21	34	55	89	144	233

$r = \frac{34}{21} \approx 1.61905$

$r = \frac{55}{34} \approx 1.61765$

$r = \frac{89}{55} \approx 1.61818$

$r = \frac{144}{89} \approx 1.61798$

$r = \frac{233}{144} \approx 1.61805$

$\boxed{r \approx 1.618}$

b) $S_{24} = \sum_{n=1}^{8} + \sum_{n=9}^{24}$

$= 1+1+2+3+5+8+13+21 + S_{24-9+1}$

$\approx 54 + \frac{34(1-1.618^{16})}{1-1.618}$

$\approx 121,378.87$

HW Questions: p. 265

EXERCISES 8.2.5 – MISCELLANEOUS QUESTIONS

1. $2k+2, 5k+1$ and $10k+2$ are three successive terms of a geometric sequence. Find the value(s) of k.

has a common ratio.

$$\frac{5k+1}{2k+2} = \frac{10k+2}{5k+1}$$

cross multiply & solve for k.

$k = 3, -\frac{1}{5} \rightarrow$ extraneous

$10\left(-\frac{1}{5}\right) + 2 = 0$

$5\left(-\frac{1}{5}\right) + 1 = 0$

2. Evaluate $\dfrac{1+2+3+\ldots+10}{1+\frac{1}{2}+\frac{1}{4}+\ldots+\frac{1}{512}} = \dfrac{S_{10}}{S_{10}}$

top → AP w/ $d=1$, first find n: $10 = 1 + 1(n-1)$

$n = 10$

$S_{10} = \frac{10}{2}[2(1) + 1(9)]$

$\boxed{\text{top} = 55}$

bottom → GP w/ $r = \frac{1}{2}$

find n → $\frac{1}{512} = 1\left(\frac{1}{2}\right)^{n-1}$

$n = 10$

$S_{10} = \dfrac{1\left(1-\left(\frac{1}{2}\right)^{10}\right)}{1-\frac{1}{2}}$

bottom $= \frac{1023}{512}$

put it together:

$= \dfrac{55}{\frac{1023}{512}}$

$\boxed{= \frac{2560}{93}}$

3. Find a number which, when added to each of 2, 6 and 13 gives three numbers in geometric sequence.

Let k = your number

$\{2+k,\ 6+k,\ 13+k, \ldots\}$

use r: $\dfrac{6+k}{2+k} = \dfrac{13+k}{6+k}$ cross multiply & solve for k.

4. Find the fractional equivalent of

(a) $2.3\dot{8}$ $2.3\overline{8}$ (b) $4.\dot{6}\dot{2}$ $4.\overline{62}$ (c) $0.41717\ldots$ $0.4\overline{17}$

$g_1 = \dfrac{8}{100}$ $r = \dfrac{0.008}{0.08} = 0.1 = \dfrac{1}{10}$

$2.3 + \underbrace{0.08 + 0.008 + 0.0008 + \ldots}_{S_\infty}$

$\dfrac{23}{10} + \dfrac{\frac{8}{100}}{1 - \frac{1}{10}}$

c) $0.4 + \dfrac{17}{1000} + \dfrac{17}{100{,}000} + $

$\dfrac{9}{9} \cdot \dfrac{23}{10} + \dfrac{\frac{8}{\cancel{100}\,10}}{} \cdot \dfrac{\cancel{10}}{9}$

$\dfrac{207}{90} + \dfrac{8}{90}$

5. Find the sum of all intergers between 200 and 400 that are divisible by 6.

204, 210, 216, ... 396 $\quad d = 6$

$396 = 204 + 6(n-1)$

$n = 33$

then $S_{33} = \frac{33}{2}[2(204) + 6(32)]$

$\frac{33}{2}(204 + 396)$

6. Find the sum of the first 50 terms of an arithmetic progression given that the 15th term is 34 and the sum of the first 8 terms is 20.

$S_{50} = ?$

$a_{15} = 34$

#1 $a_1 + 14d = 34$

$S_8 = 20$

$\frac{8}{2}[2a_1 + d(7)] = 20$

#2 $2a_1 + 7d = 5$

Solve the system for a_1 & d $\quad a_1 = -8 \quad d = 3$

then $S_{50} = \ldots$

7. Find the value of p so that $p + 5, 4p + 3$ and $8p - 2$ will form successive terms of an arithmetic progression.

1 2 3

$$4p+3-(p+5) = 8p-2-(4p+3)$$

8. For the series defined by $S_n = 3n^2 - 11n$, find a_n and hence show that the sequence is arithmetic.

$a_n = \underbrace{a_1 + d(n-1)}$

(Notice this is part of the right side)

$3n^2 - 11n = \frac{n}{2}[2a_1 + d(n-1)]$

$\frac{2}{n}(3n^2 - 11n) = \frac{2}{n}\cdot\frac{n}{2}[2a_1 + d(n-1)]$

$6n - 22 = 2a_1 + d(n-1)$

(We're close. Too many a_1 so $-a_1$)

$-a_1 \quad -a_1$

$6n - 22 - a_1 = \underbrace{a_1 + d(n-1)}$

$\boxed{6n - 22 - a_1 = a_n}$

Now find first term

$a_1 = 6(1) - 22 - a_1$

$+a_1 \quad +a_1$

$2a_1 = -16$

$\boxed{a_1 = -8}$

So: $a_n = 6n - 22 - (-8)$

$\boxed{a_n = 6n - 14}$

hence... using our a_n formula we find a_2 & a_3 then use them to show we have a common difference (proving an arithmetic sequence)

$a_2 = 6(2) - 14 = -2$

$a_3 = 6(3) - 14 = 4$

$\{-8, -2, 4, \ldots\}$

$d = 6$

9. How many terms of the series $6 + 3 + \frac{3}{2} + \ldots$ must be taken to give a sum of $11\frac{13}{16}$?

find n

$r = \frac{1}{2}$

$S_n = \frac{189}{16}$

$$\frac{189}{16} = \frac{6\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}}$$

$$\frac{189}{16} = 12\left(1-\left(\frac{1}{2}\right)^n\right)$$

$$\frac{63}{64} = 1-\left(\frac{1}{2}\right)^n$$

$$\left(\frac{1}{2}\right)^n = \frac{1}{64}$$

$$\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^6$$

$$\boxed{n = 6}$$

10. If $1 + 2x + 4x^2 + \ldots = \frac{3}{4}$, find the value of x.

$r = 2x$

$S_\infty = \frac{3}{4}$

if $|2x| < 1$

$\frac{3}{4} = \frac{1}{1-2x}$

cross multiply & solve for x

check to be sure $|2x| < 1$

11. Logs of wood are stacked in a pile so that there are 15 logs on the top row, 16 on the next row, 17 on the next and so on. If there are 246 logs in total,

(a) how many rows are there? (n)

(b) how many logs are there in the bottom row?

$15+16+17+\ldots = 246$

$d=1$

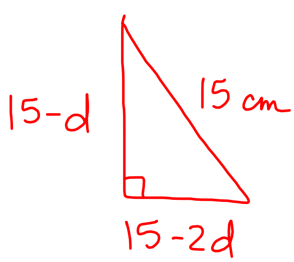
a) $\frac{n}{2}[2(15)+1(n-1)] = 246$

$n = 12$ (throw away -41 because n = positive integer)

b) $a_{12} = 15 + 1(11)$

$= 26$ logs in the bottom row.

12. The lengths of the sides of a right angled triangle form the terms of an arithmetic sequence. If the hypotenuse is 15 cm in length, what is the length of the other two sides?



$$(15-d)^2 + (15-2d)^2 = 15^2$$

13. The sum of the first 8 terms of a geometric series is 17 times the sum of its first four terms. Find the common ratio.

$S_8 = 17(S_4)$ let $g = 1^{st}$ term

$$\cancel{(1-r)} \cdot \frac{g(1-r^8)}{\cancel{1-r}} = \frac{17g(1-r^4)}{\cancel{1-r}} \cdot \cancel{(1-r)}$$

$$\frac{\cancel{g}(1-r^8)}{\cancel{g}} = \frac{17\cancel{g}(1-r^4)}{\cancel{g}}$$

$(r^4)^2$ r^4

$$1-r^8 = 17-17r^4$$

$$-16 = r^8 - 17r^4$$

$$0 = r^8 - 17r^4 + 16$$

$$0 = (r^4-16)(r^4-1)$$

$r^4 = 16$ or $r^4 = 1$

$\boxed{r = \pm 2}$ $r \neq 1$ $\left\{ \text{because of denom. original eq.} \right.$

14. Three numbers a, b and c whose sum is 15 are successive terms of a G.P, and b, a, c are successive terms of an A.P. Find a, b and c.

GP $\{a, b, c, \dots \rightarrow r \Rightarrow \boxed{\frac{b}{a} = \frac{c}{b}}$ equation #1

$\boxed{a+b+c = 15}$ equation #2

AP $\{b, a, c, \dots \rightarrow d \Rightarrow \boxed{a-b = c-a}$ eq. #3

3 equations, 3 unknowns ... solve the system.

To start: solve #2 & 3 for c, then set =

#2 → $c = 15-b-a$ → $15-b-a = 2a-b$

#3 → $c = a-b+a$ $\quad 15-a = 2a$

$c = 2a-b$ $\quad 15 = 3a$

$\boxed{a = 5}$

Now from eq. #1, cross multiply to get: $b^2 = ac$

$b^2 = 5c$

from above #3: $c = 2a-b$

$5(c) = (2(5)-b)5$

$5c = 50-5b$ → $50-5b = b^2$

Solve for b, then plug in to find c. ☺

15. The sum of the first n terms of an arithmetic series is given by $S_n = \frac{n(3n+1)}{2}$.

(a) Calculate S_1 and S_2.

(b) Find the first three terms of this series.

(c) Find an expression for the nth term.

a) $S_1 = \frac{1(3(1)+1)}{2}$ . . .

$S_2 =$ plug in 2 for n . . .

b) $a_1 = S_1 =$

$a_2 = S_2 - S_1 =$

$a_3 =$ . . .

c) $a_1 = 2 \quad d = 3$

$a_n = 2 + 3(n-1)$

$a_n = 3n - 1$

Practice:

Gabi gets paid \$100 the first week, \$125 the second week, \$150 the next and so on. Liz gets paid \$1 the first week, \$2 the next, \$4 the next, and so on.

1. When is Liz's weekly pay better?

2. When is her total pay better?

Gabi

n	1	2	3
	100,	125,	150, ...

$d=25$

Liz

n				
	1,	2,	4,	8, ...

$r=2$

$y=100+25(n-1)$

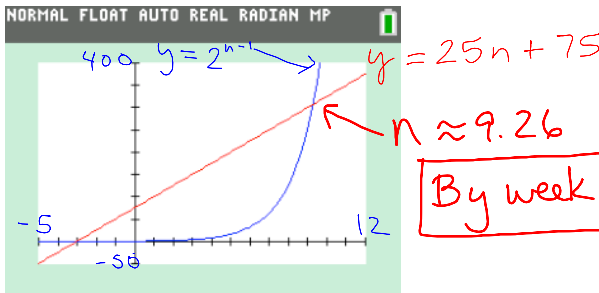
$y=25n+75$

$y=1(2)^{n-1}$

$y=2^{n-1}$

When will Liz's be greater?

$$2^{n-1} > 25n+75$$



$\boxed{\text{By week 10}}$

Gabi

100, 125, 150, ...

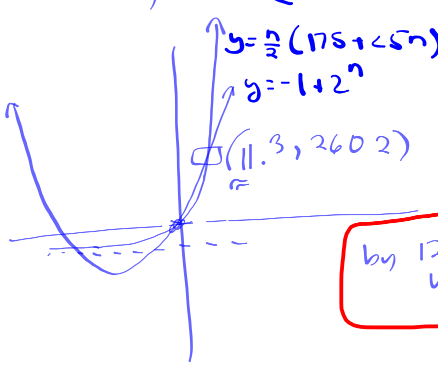
Liz

1, 2, 4, 8, ...

$$\frac{n}{2}\left[2(100)+25(n-1)\right] < \frac{1(1-2^n)}{1-2}$$

$$\frac{n}{2}\left[200+25n-25\right] < \frac{1-2^n}{-1}$$

$$\frac{n}{2}(175+25n) < -1+2^n$$



$\boxed{\text{by 12th week}}$

HW: Review WS

(will be turned in tomorrow!)

Test Tomorrow:

SL Chapter 8

Make sure you can solve equations by isolation, logarithms, and graphing.

Answers to the back of rev. ws.

4a) $y \approx \$33,598$

b) $S_{35} \approx \$1,556,899$

c) 12 years

d 27 years

5a) ≈ 1.77 m

b) 11^{th} bounce